Maximizing information in mobile sensor webs

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**Hen and chicks**

**Hen**: “bird’s eye view”; can direct chicks to gather information, collect information

**Chicks**: Chicks can collect different types of data necessary for understanding the environment
Autonomous Collaborative Search

Measure the range to the target
Objective

- Real-time system that maintains composite aerial images of specified zones using multiple UAVs
- Coordinate demands from humans and automated agents.

Multiple users simultaneously share control over UAVs and request images of subregions of particular interest to them. The system computes and revises optimal routes and schedules for UAVs.
Stanford Testbed of Autonomous Rotorcraft for Multi-Agent Control (STARMAC)
Mobile Sensor Network Control

Control Objectives:
• Automatic information gathering
• Safe interaction

Constraints:
• Power budget
• Communication bandwidth
• Computational resources
STARMAC Quadrotor Helicopters

- Testbed Purpose
  - UAV control design
  - Multi-agent control

- Testbed Composition
  - Quadrotor helicopters
  - 6 flightworthy vehicles

- Embedded Computation
  - Estimation
  - Control

- Self Sufficient UAV’s
  - Onboard computation
  - Onboard sensing
Connecting Control Actions and Sensing Capabilities
Controller goal is to minimize the uncertainty of $p(\theta)$

There may be no reason to move towards the target
Modeling Uncertainty to Increase Knowledge

Target State: $\theta$  |  Vehicle States: $x_t$
Observations: $z_{t+1}$  |  Control Inputs: $u_t$
Target State Model: $p(\theta)$  |  Motion Model: $x_{t+1} = f(x_t, u_t)$

Sensor model: $p(z_{t+1}|\theta; x_{t+1})$

Use **Bayes’ Rule** to update the target state model,

$$p(\theta|z_{t+1}; x_{t+1}) = \frac{p(\theta)p(z_{t+1}|\theta; x_{t+1})}{p(z_{t+1}; x_{t+1})}$$

Minimize the **expected future uncertainty**,

$$H(\theta|z_{t+1}) = H(\theta) - I(\theta; z_{t+1})$$
Bayes’ Rule using Particle Filters

\[ p(\theta|z_{t+1};x_{t+1}) = \frac{p(\theta)p(z_{t+1}|\theta;x_{t+1})}{p(z_{t+1};x_{t+1})} \]

- Uses available prior knowledge
- Allows multimodal posterior
- Permits nonlinear & non-Gaussian models
\[ \begin{align*}
\text{minimize} \quad & -I^{(i)}(x_t^{(i)}, u_t^{(i)}, \theta_t^{(i)} | x_t^{(-i)}, u_t^{(-i)}) \\
\text{subject to} \quad & x_{t+1}^{(i)} = f_t^{(i)}(x_t^{(i)}, u_t^{(i)}) \\
& z_{t+1}^{(i)} = h_t^{(i)}(x_{t+1}^{(i)}, \theta_t^{(i)}, \eta_t^{(i)})
\end{align*} \]

Inalhan, Stipanovic, Tomlin, CDC, 2002
Comparison of Multiple Vehicle Sim Results

Mean and Quartile Error Bars for 1000 Trials

Probability of Target being < 1 meter from Estimate

Time Elapsed
Optimality for Linear Approximation

The goal is to minimize uncertainty (Shannon entropy):

$$\text{minimize } \log |\Sigma_t|$$

Equivalently, we can maximize information:

$$\text{maximize } \log |\Omega_t|$$

because $\Omega_t = \Sigma_t^{-1} \Rightarrow |\Omega_t| = |\Sigma_t|^{-1} \Rightarrow \log |\Omega_t| = -\log |\Sigma_t|$

Use the standard information matrix update:

$$\Omega_t = \Omega_{t-1} + \sum_{v=1}^{n_v} H_v^T \sigma_r^{-1} H_v$$
Range-Only Measurement Model

Sensor Model
\[ \theta_{t/v} = \arctan \left( \frac{y_t - y_v}{x_t - x_v} \right) \]
\[ z = \sqrt{(x_t - x_v)^2 + (y_t - y_v)^2} + \nu \]
\[ \nu \sim \mathcal{N}(0, \sigma_r) \]

Sensor Jacobian w.r.t. Target
\[ H = \frac{1}{r} \left[ \begin{array}{c} -(x_t - x_v) \\ -(y_t - y_v) \end{array} \right] \]
\[ = \left[ \begin{array}{c} -\cos(\theta_{t/v}) \\ -\sin(\theta_{t/v}) \end{array} \right] \]

Sensor Information Contribution
\[ H^T \sigma_r^{-1} H = \sigma_r^{-1} \left[ \begin{array}{cc} \cos^2(\theta_{t/v}) & \frac{1}{2} \sin(2\theta_{t/v}) \\ \frac{1}{2} \sin(2\theta_{t/v}) & \sin^2(\theta_{t/v}) \end{array} \right] \]
Range-Only Information

Using the Jacobian for the Range-Only sensor,

\[ \Omega_t = \Omega_{t-1} + \sigma_r^{-1} \sum_{v=1}^{n_v} \begin{bmatrix} \cos^2(\theta_{t/v}) & \frac{1}{2}\sin(2\theta_{t/v}) \\ \frac{1}{2}\sin(2\theta_{t/v}) & \sin^2(\theta_{t/v}) \end{bmatrix} \]

Information is not a function of range, so the optimization is:

\[ \text{maximize } \log |\Omega_t| \]

\[ \theta_{t/1}, ..., \theta_{t/n_v} \]

Solving analytically, the necessary and sufficient conditions for optimal sensing locations (for \( \Omega_t = kI \)) are:

\[ \sum_{v=1}^{n_v} \cos(2\theta_{t/v}) = 0 \quad \sum_{v=1}^{n_v} \sin(2\theta_{t/v}) = 0 \]
Optimal Range-Only Configurations

2 Vehicles

4 Vehicles

5 Vehicles

3 Vehicles

Note: Solution assumes prior information equal in x and y directions
Information Gain vs. Sensing Location

Effect of multiple vehicles

- Equal x and y prior Info
- **One** Other Vehicle
- Equal x and y prior Info
- **Two** Other Vehicles
- Prior info strong in x
- **Two** Other Vehicles

![Graphs showing effect of multiple vehicles with different prior information and vehicle configurations.](image)
Particle Filter Information Gain

- Initial distribution is uniform over a square
- Range “ring” curvature makes range matter near the target
- Steady state is similar to Gaussian
Range-Only Example

Measure the distance to the target
Bearings-Only Example

Measure the **direction** to the target
Beacon Field Example

Measure the field line orientation
Research Directions

- Flight Tests of Autonomous Search
- Humans sharing control with automation
- Generalize to Other Applications
  - Unexploded ordinance detection
  - Submarine detection
  - Beacon tracking scenarios
  - RFID tracking
  - Survey of disaster areas
  - Biological studies, animal monitoring
- Sensor Model Development, p{detection}
- Specialized Optimization Algorithms
- Reachable Set-Based Collision Avoidance Development
Three vehicle flight
Angular accelerations and vertical acceleration are controlled by varying the propeller speeds.
Pairwise Cooperation

Iterative Distributed Optimization: Agents agree on control actions to take through an iterative algorithm.

Decoupled Optimization: Agents independently determine their best control actions with respect to the group.
Probability of Target being < 1 unit from Estimate

Mean and Quartile Error Bars for 1000 Trials

- Pairwise Approximation
- Solo Approximation

Time Elapsed

0.1

0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9

1
Bearings-Only Measurement Model

Sensor Model

\[
\theta_{t/v} = \arctan \left( \frac{y_t - y_v}{x_t - x_v} \right)
\]

\[
z = \theta_{t/v} + \nu
\]

\[
\nu \sim \mathcal{N}(0, \sigma_b)
\]

Sensor Jacobian w.r.t. Target

\[
H = \frac{1}{r_v^2} \left[ (y_t - y_v) - (x_t - x_v) \right]
\]

\[
= \frac{1}{r_v} \left[ \sin(\theta_{t/v}) - \cos(\theta_{t/v}) \right]
\]

Sensor Information Contribution

\[
H^T \sigma_b^{-1} H = \sigma_b^{-1} \frac{r_v^2}{r_v} \left[ \begin{array}{cc}
\sin^2(\theta_{t/v}) & -\frac{1}{2} \sin(2\theta_{t/v}) \\
-\frac{1}{2} \sin(2\theta_{t/v}) & \cos^2(\theta_{t/v})
\end{array} \right]
\]
Using the Jacobian for the Bearing-Only sensor,

\[
\Omega_t = \Omega_{t-1} + \sigma_b^{-1} \sum_{v=1}^{n_v} \frac{1}{r_v^2} \begin{bmatrix}
\sin^2(\theta_{t/v}) & -\frac{1}{2} \sin(2\theta_{t/v}) \\
-\frac{1}{2} \sin(2\theta_{t/v}) & \cos^2(\theta_{t/v})
\end{bmatrix}
\]

The global solution is trivial – the origin

Constraining all vehicles to be at the same radius, the optimal sensing locations are the same as for range-only, with equal prior information in both directions:

\[
\sum_{v=1}^{n_v} \cos(2\theta_{t/v}) = 0 \quad \sum_{v=1}^{n_v} \sin(2\theta_{t/v}) = 0
\]
Information Gain vs. Sensing Location

Effect of prior information

- Equal x and y prior Info
- No Other Vehicles

- Prior info strong in x
- No Other Vehicles
Information Gain vs. Sensing Location

Effect of multiple vehicles

- Ignoring both vehicles
  - Equal x and y prior Info

- Including one Vehicle
  - Equal x and y prior Info

- Including both vehicles
  - Equal x and y prior Info
Particle Filter Information Gain

• Initial distribution is uniform over a square
• Steady state is similar to Gaussian
• Differences from the Gaussian case are minor improvements
• More consistent with the Gaussian case than the range-only sensor
• Advantage lies more in ability to directly use a particle filter
Rescue Beacon Measurement Model

Sensor Model

\[ \theta_{v/t} = \arctan \left( \frac{y_v - y_t}{x_v - x_t} \right) \]
\[ \alpha = \theta_{v/t} - \psi_t \]
\[ z = \theta_{v/t} - \arctan (2 \cot(\alpha)) + \nu \]
\[ \nu \sim \mathcal{N}(0, \sigma_\nu) \]

Note: Signal strength may provide useful information as well, though not as reliably.
Rescue Beacon EIF

Sensor Jacobian w.r.t. Target (for $\hat{\psi}_t = 0$)

$$H = \frac{1}{3x^2 + r^2} \left[ -\frac{\sin(\theta_{t/v})}{r} \left( 3x^2 - r^2 \right) \frac{\cos(\theta_{t/v})}{r} \left( 3x^2 - r^2 \right) 2r^2 \right]$$

Sensor Information Contribution

$$H^T \sigma_a^{-1} H = \frac{\sigma_a^{-1}}{(3x^2 + r^2)^2} \begin{bmatrix} \frac{\sin^2(\theta_{t/v})}{r^2} (3x^2 + 3r^2)^2 & -\frac{\sin(2\theta_{t/v})}{2r^2} (3x^2 + 3r^2)^2 & -2r \sin(\theta_{t/v}) (3x^2 + 3r^2) \\ -\frac{\sin(2\theta_{t/v})}{2r^2} (3x^2 + 3r^2)^2 & \frac{\cos^2(\theta_{t/v})}{r^2} (3x^2 + 3r^2)^2 & 2r \cos(\theta_{t/v}) (3x^2 + 3r^2) \\ -2r \sin(\theta_{t/v}) (3x^2 + 3r^2) & 2r \cos(\theta_{t/v}) (3x^2 + 3r^2) & 4r^4 \end{bmatrix}$$

- Best info for $\hat{x}_t$ close to $y$-axis, near the target mean
- Best info for $\hat{y}_t$ close to $x$-axis, near the target mean
- Best info for $\hat{\psi}_t$ along the $y$-axis
Effect of orientation uncertainty

- **High** $\psi$ prior info
- Equal $x$ and $y$ prior Info
- No Other Vehicles

- **Low** $\psi$ prior info
- Equal $x$ and $y$ prior Info
- No Other Vehicles
Information Gain vs. Sensing Location

The available information depends on the prior information

- **Equal** $x$ and $y$ prior Info
  - No Other Vehicles

- **Prior info strong in** $x$
  - No Other Vehicles

- **Prior info strong in** $y$
  - No Other Vehicles
Information Gain vs. Sensing Location

Effect of multiple vehicles

- Ignoring both vehicles
  - Equal $x$ and $y$ prior Info

- Including one Vehicle
  - Equal $x$ and $y$ prior Info

- Including both vehicles
  - Equal $x$ and $y$ prior Info
Particle Filter Information Gain

• Initial distribution is uniform over a square
• Differs from linear approximation when uncertainty is large
• Uncertainty is large for much of simulation
• Particle Filters can capture the structure of the more complicated distribution