Outline

- Model
  - *Games with continuous state spaces* in which players communicate with messages through *faulty channels*.

- Theorems
  - Convergence to *Nash equilibria* for this model

- Applications
  - *Pattern formations* of mobile agents with faulty channels

- Current and Further work
  - Exploring applications to other areas, e.g., economics and biology
Communication Models

- **Synchronous Shared-state**
  - Instantaneous communication, Rounds

- **Asynchronous via Message-passing**
  - Messages may be *lost, delayed, duplicated* or *received out of order*
Communication Medium

- **Broadcast Channel**
  - Agents: send, receive
  - Internal Actions: duplicate, drop

- **Assumptions**
  - **Bounded Delay**: messages are eventually dropped or received
  - **Bounded Duplication**: total number of copies is finite
  - **Fairness Communication**: For all $i, j$: if $j$ sends infinitely often then $i$ receives messages from $j$ infinitely often
Basic Question

- Can we discharge proofs of MP by using the following theorem:
  
  “If SS satisfies a specification then MP also satisfies the same specification provided that…”?

- Agents update their positions using the state of the other agents at some unknown time in the past.
Best Response Games with Concurrent Moves

- **Best Response** function $\beta_i : S^{N-1} \rightarrow S$
- **Player $i$ Move**: set its value to its Best Response

![Diagram](image-url)
Best Response Games with Alternating Moves

- Players alternate their moves
Best Response Games via Message-passing

- **Player $i$:** sets its value to its best response
Better Response Games via Message-passing & Dynamics

- Player $i$: moves towards its best response
Stable and Unstable Nash Equilibrium

Stable Equi. Point

Unstable Equi. Point

$P_1$ $P_2$
Proving Convergence

- Show a (Lyapunov) function that is non-increasing along all executions of the system.

- Show a collection of Sets \( \{ R_i \}_{i \in \mathbb{N}} \) satisfying:
  - C1. Monotonicity
  - C2. Initial States
  - C3. Stability
  - C4. Progress
Proving Convergence of MP using Independent Lyapunov Functions

- **Theorem.** If there exists Lyapunov functions $R$ that satisfy C1-C4 where

  \[ R = r_1 \wedge r_2 \wedge \ldots \wedge r_N \]

  then if $SS$ converges to $s^*$, then $MP$ converges to $s^*$. 

Predicate on the state of agent $i$.
From Independence to Rectangular Level Sets

Initial States

Equilibrium

$s^*$
Why Independence?

- **Non-independence**

All possible trajectories with delayed messages remain inside the rectangular level sets!
Solving Systems of Linear Equations via MP

- **Goal**: $Ax = b$
- A invertible
- **Iterative Methods**.
  - E.g. Jacobi
    \[ x_i(t) = b_i - \sum_{j \neq i} a_{ij} x_j(t-1) \]
    *Linear Best Response Function*

- What about *PS* ?!?!
  - Agent $i$ solves the equation with $t_{\tau j} < t$
    \[ x_i(t) = b_i - \sum_{j \neq i} a_{ij} x_j(t_{\tau j}) \]
Rectangular Level Sets

**Proposition.** If $A$ is *weakly diagonally dominant*, then the $PS$ system converges to $A^{-1} \cdot b$

- Weakly Diagonally Dominant
  - $\forall j : \sum_{k \neq j} |A[j,k]| \leq 1$
  - $\exists j : \sum_{k \neq j} |A[j,k]| < 1$
Application to Pattern Formation

- Local Rule: average of their neighbors
Rectangular Level Sets

Left Profile

error

0 1 2 3 4 5 6
Current and Future Work

- Mixed Equilibria with Stochastic Systems
- Applications in
  - Economics: Cournot’s Oligopoly. Competition among Producers with Faulty Communication
  - Biology: Ant Hills.
Cournot’s Oligopoly

- Competition among Producers with Faulty Communication
Results for Cournot’s Oligopoly

- **Proposition.** Converge despite faulty communication provided that the infinity norm of the gradient of the best response functions is less than unity.

\[ \forall j \; | \nabla \beta_j (x_j) |_\infty < 1 \]
Related Work

- **Distributed Control**
  - Murray, Olfati, 2007, Jadbabaie, Lin, and Morse, 2003

- **Computer Science**
  - Dolev et al. 2007, 2009
  - Hendriks 2005, Lamport 2005

- **Economics**
  - Gabay and Moulin 1980
  - Takahashi and Wen, 2003

- **Computational Mathematics**
  - Chazan and Miranker, 1969

- **Biology**
References

