Wireless Control of Passive Systems Subject to Actuator Constraints

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Background on Passivity

- Passive systems sometimes are Lyapunov Stable
- Strictly Output Passive Systems are $L^m_2 (l^m_2)$ stable.
  - When connected in either a parallel or negative feedback manner the overall system remains passive
- Passive control theory applies to
  - Linear and nonlinear systems
  - Continuous and discrete-time systems
- Straight forward to propose controllers for:
  - Independent joint PD controller for robotic manipulator
- Cascades of Passive Systems Typically are Not Passive
  - Memoryless Nonlinearity $\rightarrow$ Passive System (not passive)
Background on Passivity

- Further reading:
  - Time Domain and Frequency Domain Conditions For Passivity
    https://wiki.isis.vanderbilt.edu/CPS/index.php/Publications

\[
\begin{align*}
\text{Passive} & \iff \text{Positive Real} \iff \text{Lyapunov Stable} \\
\frac{K}{s} & \iff \frac{K}{s^2 + a^2} \\
\text{Strictly Input Passive (SIP)} & \iff \frac{K(s+a)}{s} \\
\text{Strongly Positive Real} & \iff \frac{s+a}{s+b} \\
\text{Strictly Output Passive (SOP)} & \iff \frac{b}{s+b} \\
\text{(Passive + Non Expansive (NE))} & \iff \text{(Passive + Asymptotically Stable)} \\
\frac{K\frac{\omega_n^2 s}{s^2 + 2\zeta \omega_n s + \omega_n^2}}{s^2 + 2\zeta \omega_n s + \omega_n^2} & \iff \frac{K\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\end{align*}
\]
Background on Passivity

- A bit more precise:
  
  **Notation:**
  
  \[ y, x \in \mathbb{R}^m \quad T = [0,1,\ldots,\infty], \quad [0,\infty] \]
  
  \[ y_T = y(t) \quad \text{if} \quad 0 \leq t \leq T, \quad 0 \quad \text{otherwise.} \]
  
  \[ y_T = y(i) \quad \text{if} \quad 0 \leq i < T, \quad 0 \quad \text{otherwise.} \]
  
  \[ \langle y, u \rangle_T = \sum_{i=0}^{T-1} y^T[i]u[i] \quad \text{or} \quad \langle y, u \rangle_T = \int_0^T y^T(t)u(t)\,dt \]

- The extended spaces \( \mathcal{H}_e \):
  
  \[ \langle x, x \rangle_T = \int_0^T x^T(t)x(t)\,dt < \infty, \quad \forall T \in T. \quad \mathcal{H}_e \equiv L^m_{2e} \]
  
  \[ \langle x, x \rangle_T = \sum_{i=0}^{T-1} x^T[i]x[i] < \infty, \quad \forall T \in T \quad \mathcal{H}_e \equiv l^m_{2e} \]
Background on Passivity

- Definition 1: Let $G : \mathcal{H}_e \mapsto \mathcal{H}_e$ then for all $u \in \hat{\mathcal{H}}_e$ and all $T \in T$:
  - $G$ is passive if there exists $\beta > 0$, s.t. $\langle Gu , u \rangle_T \geq -\beta$ holds.
  - $G$ is strictly-output passive if there exists $\beta , \epsilon > 0$, s.t. $\langle Gu , u \rangle_T \geq \epsilon \| Gu_T \|^2 - \beta$ holds.
  - $G$ is strictly-input passive if there exists $\beta , \delta > 0$, s.t. $\langle Gu , u \rangle_T \geq \delta \| u_T \|^2 - \beta$ holds.
- Simple to construct such as Euler-Lagrange systems:
  \[
  \tau = M(\Theta)\ddot{\Theta} + C(\Theta, \dot{\Theta})\dot{\Theta} + g(\Theta) \\
  M(\Theta) = M(\Theta)^T > 0 \quad x^T(M - 2C)x = 0. \forall x \in \mathbb{R}^n. \\
  \int_0^T \dot{\Theta}(t)^T \tau_u(t) dt = V(x(T)) - V(x(0)) \\
  \int_0^T \dot{\Theta}(t)^T \tau_u(t) dt \geq -V(x(0)).
  \]
Inner-product Equivalent Sample and Hold (IPESH)

\[
\sum_{i=0}^{N-1} y^T[i]u[i] = \int_0^{NT_s} y^T(t)u(t)\,dt
\]

\[y(i) = (z-1)\]

\[
G_{ct} \quad G_d(u(i)) : u(i) \rightarrow y(i)
\]

I. \(x(t) = \int_0^t y(\tau)\,d\tau\)

II. \(y[i] = x[(i + 1)T_s] - x[iT_s]\)

III. \(u(t) = u[i], \forall t \in [iT_s, i(T_s + 1)]\)
Theorem 1, Using the IPESH:

I. if $G_{ct}$ is passive, then $G_d$ is passive,

II. if $G_{ct}$ is strictly-input passive, then $G_d$ is strictly input passive

III. if $G_{ct}$ is strictly-output passive, then $G_d$ is strictly output passive

* Theorem 1-III, correction from CDC 2007
Passivity Based $m_2$ stable digital control net.

- $G_p$ and $G_c$ are passive, $K_p > 0$, $K_c > 0$, no duplicate wave variables processed $\rightarrow$ strictly-output passive ($m_2$-stable) network.

$u \in l^2(U) \Rightarrow G(u) \in l^2(Y)$

$\| (G(u))_N \|_2 \leq \gamma \| u_N \|_2 + b, \forall u \in l^2_e(U)$
Fundamental Problem: Actuator Constraints (memoryless nonlinearities) Prevent Passivity

Theorem 2: Assume there exists an input-output pair \((u(x), y(x))\) in which there exists \(\epsilon < 0\), and \(\delta > 0\), \(T > 0\) \((N > 1)\)

\[\alpha = 1\] and index \(j\) s.t.:

\[\alpha \epsilon = \int_{T-\delta}^{T} u_j(t)y_j(t)dt\quad \text{where } u_j(t) = \alpha k_j u_{\text{max},j} \text{sign}(u_j(T-\delta)) \text{ or}
\]

\[\alpha \epsilon = u_j[N-1]y_j[N-1]\quad \text{in which } u_j[N-1] = \alpha k_j u_{\text{max},j} \text{sign}(u_j[N-1]).\]

then the cascaded system is not passive, otherwise it is passive.
Fundamental Problem: Actuator Constraints (memoryless nonlinearities) Prevent Passivity

\[ G(u(s)) = \frac{1}{s+1} \]
Memoryless Non-linearities

ASSUMPTION

\[ \sigma : \mathbb{R}^m \rightarrow \mathbb{R}^m \text{ and } \forall i \in \{1, 2, \ldots, m\}, \]
\[ \text{if } u_i(x) = 0 \text{ then } \sigma_i(u(x)) = 0 \]
A non-linear controller


Definition 3: The inner-product recovery block (IPRB) $\beta(u(x))$ has the following form: $\beta(u(x)) = \text{diag}(\beta_1(u(x)), \ldots, \beta_m(u(x)))$, where

$$\beta_i(u(x)) = \begin{cases} \frac{\sigma_i(u(x))}{u_i(x)}, & \text{if } u_i(x) \neq 0 \\ 1, & \text{if } u_i(x) = 0 \end{cases}$$

(41)

See proof for Lemma 1
Proposed Architecture

Passive System

Wave Variables

Inner-Product Recovery Block

Memoryless Nonlinearity

SOP Plant

Wireless Network

SOP Digital Controller
Main Result

Theorem 4: The digital control network depicted in Fig. 1 in which \( K_p > 0, K_c > 0, G_c \) are passive and the passive plant \( G_p \) is subject to memoryless nonlinearities \( \sigma(\cdot) \), is strictly-output passive which is sufficient for \( l^m_2 \)-stability if

\[
\langle f_{op}, e_{doc}\rangle_N \geq \langle e_{oc}, f_{opd}\rangle_N
\]

(40)

holds for all \( N \geq 1 \).

How does the inner-product recovery block preserve passivity? Are any additional passivity properties preserved using the inner-product recovery block if we further restrict \( \sigma(\cdot) \)?
Theorem 3: Using the IPRB, the following can be said about,

\[ H_r : u(x) \mapsto y(x) \text{ given } G : \sigma(u(x)) \mapsto p(x) \]

I. If \( G \) is passive then \( H_r \) is passive.

II. If \( G \) is strictly-output passive then \( H_r \) is strictly-output passive if \( \sigma_{\text{MAX}}(\beta(u))^2 < \infty, \forall u \in \mathbb{R}^m \)

III. If \( G \) is strictly-input passive then \( H_r \) is strictly-input passive if there exists a \( \gamma > 0 \) such that \( \sigma^T(u)\sigma(u) \geq \gamma u^T u, \forall u \in \mathbb{R}^m \).
Corollary 1: Using the continuous time IPRB and IPESH the following can be said about, $G : \sigma(u(t)) \leftrightarrow p(t)$ and $H_{rd} : u(i) \leftrightarrow y(i)$

I. If $G$ is passive then $H_{rd}$ is passive for all $\sigma(u)$.

II. If $G$ is strictly-input passive then $H_{rd}$ is strictly-input passive if $\sigma_{MIN}(\beta(u))^2 > 0$, $\forall u \in \mathbb{R}^m$.

III. If $G$ is strictly-output passive then $H_{rd}$ is strictly-output passive if $\sigma(u)$ is a sector $[k_1, k_2]$ nonlinearity such that $\sigma_{MAX}(\beta(u))^2 = \max(k_1^2, k_2^2) < \infty$. 

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Discrete Time IPRB

Corollary 2: Using the discrete time IPRB and IPESH the following can be said about, $G : \sigma(u(t)) \leftrightarrow p(t)$ and $H_{dr} : u(i) \leftrightarrow y(i)$.

I. If $G$ is passive then $H_{dr}$ is passive for all $\sigma(u)$.

II. If $G$ is strictly-input passive then $H_{dr}$ is strictly-input passive if $\sigma_{MIN}(\beta(u))^2 > 0$, $\forall u \in \mathbb{R}^m$.

III. If $G$ is LTI and strictly-output passive then $H_{dr}$ is strictly-output passive if the sector $[k_1, k_2]$ nonlinearity is such that $\sigma_{MAX}(\beta(u))^2 = \max(k_1^2, k_2^2) < \infty$. 

$\sigma(\text{ZOH}(u(i))) = \sigma(u(t)) = \text{ZOH}(\sigma(u(i)))$
sector\([k_1, k_2]\) nonlinearity?
Definition 4: A memoryless nonlinearity $\sigma: \mathbb{R}^m \rightarrow \mathbb{R}^m$ is said to satisfy a global sector condition belonging to sector $[k_1, k_2]$ if

$$k_1 u^T u \leq u^T \sigma(u) \leq k_2 u^T u, \ \forall u \in \mathbb{R}^m$$

holds any $k_1, k_2 \in \mathbb{R}$. If (44) holds with strict inequality, then $\sigma(\cdot)$ is said to belong to a sector $(k_1, k_2)$. 
sector\([k_1,k_2]\) nonlinearity?

**Theorem 5:** Let \(k_1, k_2 \in \mathbb{R}\) with \(k_1 \leq k_2\). Let \(\sigma : \mathbb{R}^m \rightarrow \mathbb{R}^m\) with \(\sigma_i(u_i = 0) = 0, i \in \{1, \ldots, m\}\) and \(\beta(u)\) as defined by Definition 3 in Appendix I. Such that the following statements are equivalent:

i) \((k_1 I - \beta(u)) \leq 0\) and \((\beta(u) - k_2 I) \leq 0, \forall u \in \mathbb{R}^m\)

ii) \(k_1 u^T u \leq u^T \sigma(u) \leq k_2 u^T u, \forall u \in \mathbb{R}^m\)

**Theorem 6:** For a given sector \([k_1, k_2]\) nonlinearity:

i) If either \(k_1 = 0\), or \(k_2 = 0\) then \(\sigma_{\text{MIN}}(\beta(u))^2 = 0\).

ii) \(\sigma_{\text{MAX}}(\beta(u))^2 = \max(k_1^2, k_2^2)\).
Conclusions

- Systems which consists of actuator saturation in cascade with a passive plant are typically not passive.
- The inner-product recovery block, preserves passivity.
- Using the inner-product recovery block we can create $l^m_2$ stable digital control networks
- If we further restrict the memoryless non-linearity to a given sector, additional passivity properties can also be preserved.
- We propose a slightly weaker sector definition, typically seen in the literature, except...
Current and Future Work

- Is there a better sector definition for MIMO?


https://wiki.isis.vanderbilt.edu/CPS/index.php/Publications
A better sector definition?

Definition 3: Assuming that $H u(0) = y(0) = 0$, then a dynamic system $H : \mathcal{H}_e \rightarrow \mathcal{H}_e$ is (strictly) inside the sector $[a, b]$, $b > 0$, $a \leq b$, $\epsilon > 0$ if

$$\|y_T\|_2^2 - (a + b) \langle y, u \rangle_T + ab \|u_T\|_2^2 \leq 0 \quad (\leq -\epsilon \|u_T\|_2^2) \quad (7)$$

- Further plans for simulation and evaluation
- Questions?